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Supply and demand in Kaldorian growth models: a proposal for dynamic adjustment

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Abstract

This paper analyses the dynamic adjustment of supply and demand in Kaldorian growth models. We discuss how the growth rate of a country given by the demand constraints may adjust towards the growth rate given by the supply-side (and vice-versa), presenting the necessary conditions for this adjustment. Our main conclusion is that, for a monopolistic economy, where firms invest to maintain a constant level of capital utilization, there are no capital constraints and hence the degree of capacity utilization is not affected by this adjustment. Nevertheless, depending on specific conditions, an economy may face labour constraints, and thus an adjustment mechanism is necessary. The Palley-Setterfield approach for this dynamic adjustment brings a possible reconciliation to supply- and demand-side long-term growth rates. It emphasizes the endogeneity of the income-elasticities of demand for imports and the Verdoorn coefficient to utilization capacity. However, some considerations about labour market have to be discussed in order to understand the characteristics and limitations of this approach. In this sense, we draw from the McCombie (2011) critique, in which employment adjusts immediately to guarantee equilibrium between supply and demand. We propose reconciliation between the Palley-Setterfield and the McCombie approaches, and present a model with a labour market adjustment in which both types of adjustments represent extreme cases, discussing the existence and the characteristics of intermediate cases.

Keywords: economic adjustment, demand-led growth, natural rate of growth, Kaldorian growth models

JEL: E12; F43; O41.

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1. Introduction

One of the most important issues in economic theory is why growth rates differ between countries and regions. On the one hand, the neoclassical and endogenous growth theories asserts (Romer, 1994; Solow, 1956) that the explanation for the differences between countries' growth rates is related to availability of factors and their allocation, which characterises a supply-oriented approach. On the other hand, the Post-Keynesian perspective (Blecker & Setterfield, 2019) emphasises the relevance of effective demand as a primary drive of accumulation, and thus the long-run growth rate is demand-driven.

Although classical economists have contributed significantly to understanding the dynamics behind the adjustment mechanisms of supply and demand, the first macroeconomic model which has explicitly provided a theory of economic growth was developed by Harrod (1939) and Domar (1947). Their models aimed at answering the same basic question: what must be the investment and saving growth rates capable of maintaining a growing economy in equilibrium, and what is the economic growth rate compatible with that? In short, the authors were looking for an explanation on why demand and supply growth rates diverge, searching for their determinants. As the Harrod-Domar growth model could not consider explicitly a self-adjustment mechanism, it opened the floor for closures both on the demand-oriented models developed by the Post-Keynesian economist as well as on the supply-oriented models developed by neoclassical economists. In the neoclassical models, countries' long term growth are explained by the supply factors (rate of growth of labour force and its productivity), and demand responds passively. As a consequence of the assumption of diminishing returns to capital (Solow, 1956), neither the investment nor the savings to GDP affected the countries' growth rates (they affected only the GDP level).

In the late 1980s, Romer (1986) and Lucas (1988) have criticized the "old" growth models arguing against the diminishing returns to capital assumption. This critique was the basis for the "new" growth models, which assumed that productivity growth is determined endogenously by the accumulation of specific factors of production. This assumption has shifted the focus of the neoclassical models from the technological change to the externalities generated by the process of factor accumulation. According to Dutt (2001, 2006), however, the neoclassical models and the "new" growth models ignore the issues related to effective demand because they assume clear labour markets and that all savings are invested (and thus the economy is always in full employment). As a result of that, the long-run growth in these models is determined exclusively by the supply factors.

Post-Keynesian growth models (Blecker & Setterfield, 2019; Harcourt & Kriesler, 2013), on the other hand, stressed the importance of demand on explaining the differences between countries' growth rates. According to Kaldor (1966), although some changes in demand have their origin on changes in supply, it is mainly supply that adjusts to demand. Countries' growth is then primarily governed by the growth of effective demand, instead of resource-constrained. However, the explanation for the adjustment mechanism is controversial. From a production perspective, a faster growth rate of demand increases productivity (via Verdoorn's law) which increases the natural rate of growth at a specific rate. From the demand perspective, growth is constrained by consumption, investment and net exports, and thus the rate of growth may be different from the natural rate. Nevertheless, these growth rates need to converge; otherwise there will be ever-growing of excess capacity or supply constraints in the long-run, which is not economically plausible. Thereby, after three-quarters of century since Harrod first published his paper on the dynamics of supply and demand, there is still a lack of consensus in the economic theory about the central drivers of economic growth.

The aim of this paper is to analyse this dynamic adjustment of supply and demand based Kaldorian supply and demand models based on the recent literature on the topic (Blecker, 2013; McCombie, 2011; Palley, 2003; Setterfield, 2006, 2011, 2013). Our aim is to discuss how growth rate of a country given by the demand constraints may adjust towards the growth rate given by the supply-side (and vice-versa), presenting the necessary conditions for this adjustment to take place considering a stable condition for employment and capacity utilization. Our main conclusion is that a monopolistic economy, where firms invest to maintain a constant level of capital utilization, has no capital constraints and thus the degree of capacity utilization does not change in the long run. However, depending on specific conditions, an economy may face labour constraints, and thus we need an adjustment mechanism through employment. The Palley-Setterfield approach brings a possible adjustment mechanism. We argue, however, that this mechanism depends on some considerations about labour market, starting from the critique by McCombie (2011).

The contribution on this paper is twofold. First, we propose and organization of the recent literature and the debates on the adjustments between supply and demand Kaldordian models in a unified framework. We explicitly raise the issue between the behaviour of labour supply and labour demand in the adjustment, as the determinants of a stable employment dynamics. Second, the output of this paper is a general model that not only deal with the different streams of the debate (Palley-Setterfield and McCombie), but also represent intermediate adjustments, in which labour supply and labour demand adjust at different rates, defining different steady state conditions.

The paper is divided in five sections. After this introduction, Section 2 presents the macro-dynamics of supply and demand adjustments based on Palley-Setterfield controversy (Palley, 2003; Setterfield, 2006), as well as McCombie's (2011) critique. Section 3 presents Setterfield (2013) argument for the need of a supply-side of Kaldorian growth models, highlighting the importance of capital and labour constraints. Section 4 presents an alternative approach for the adjustment mechanisms based on Setterfield's (2013) argument, as well as the necessary conditions a reconciliation of supply and demand. Finally, in the last section, we conclude the paper.

2. The macro-dynamics of supply and demand adjustment

In this section we organize the literature around the adjustments on Kaldorian growth models.

2.1. Reconciling supply and demand: Palley's pitfall

With the aim of reconciling supply and demand growth rates, Palley (2003) argues that, if demand and supply growth rates are not the same, "there will either be growing excess of capacity or growing excess demand – neither of which are observed in capitalist economies". Thereby, the natural growth rate and the actual growth rate have to converge in the long run.

The natural growth rate, which provides the supply-side from a Kaldorian perspective, is given by:

$$y_N = l + q = l + \lambda + vy \tag{2.1}$$

In which y_N and y are, respectively, the natural and actual growth rates, l is the labour force growth rate, q is the growth rate of productivity, λ is the exogenous technical change, and v is the Verdoorn's coefficient (the sensibility of productivity growth to actual growth rate).

Palley assumes the Balance-of-Payment Constrained Growth (BPCG) model to analyse the demand-side. Hence, the actual growth rate is given by Thirlwall's law[3]:

$$y = y_B = \frac{\varepsilon}{\pi} z \tag{2.2}$$

where y_B is the BPCG rate, ε and π are the income elasticities of demand for exports and imports, respectively, and z is the world growth rate.

Re-arranging equation (2.1), and considering that natural growth rate has to be equal to the actual growth rate (which means that an economy is not facing either growing excess of capacity or growing excess demand), a necessary condition for an automatic adjustment of supply and demand sides of Kaldorian models is that:

$$y_B = \frac{l + \lambda}{1 - v} \Rightarrow z = \frac{(l + \lambda)\pi}{(1 - v)\varepsilon} \tag{2.3}$$

However, once there is nothing in the BPCG models that ensures that this fact occurs, Palley concludes that the model is over-determined. It depends on this specific situation for the world growth rate to avoid a growing imbalance between actual and capacity output.

[3] Although the effective growth rate should be given by the sum of the aggregate demand macroeconomic variables, following Palley (2003) and Setterfield (2006, 2011), we use the Thirlwall's law equation as the actual growth rate. The actual growth rate needs to converge to the one compatible with balance-of-payments constraints, otherwise the economy goes out of bounds in terms of its net exports - see Porcile & Spinola (2018).

With the aim of solving this issue, the author suggests that the income elasticity of demand for imports is negatively related to the excess of capacity utilization. According to Palley (2003), the "rationale for this is that imports are driven by bottlenecks. As the rate of capacity and employment decrease, bottlenecks become more prevalent and the share of increments in income spent on imports increases"(p. 80). Hence, it follows that:

$$\pi = \pi(E), \pi' > 0 \tag{2.4}$$

In which E is defined as the degree of capacity utilization – it can be interpreted equally as the degree of labour utilization and the degree of capital utilization).

In this vein, Palley stressed that the natural growth rate affects the income elasticities of demand, and thus BPCG responds passively to changes in the natural rate of growth[4]. If $y > y_N$, the level of capacity utilization, E, grows and the income elasticity of demand for imports rises. As a consequence, BPCG rate decreases and the supply and demand growth rates return to equilibrium in a situation in which the capacity of utilization is higher than it was in the starting point. Essentially, based on equation (2.2), we have that:

$$\frac{\partial y_B}{\partial E} = \frac{\partial y_B}{\partial \pi}\frac{\partial \pi}{\partial E} = -\frac{\varepsilon}{\pi^2} z \frac{\partial v}{\partial E} \tag{2.5}$$

Because $\frac{\partial v}{\partial E} > 0$ and all other variables are positive, an increase in the degree of capacity utilization affects negatively the actual rate of growth (given by Thirlwall's law), and thus it adjusts towards the natural rate of growth.

The impact of changes in the degree of capacity utilization on the natural rate of growth, on the other hand, is described by:

$$\frac{\partial y_N}{\partial E} = \frac{\partial y_N}{\partial y}\frac{\partial y}{\partial E} = v\frac{\partial y}{\partial E} \tag{2.6}$$

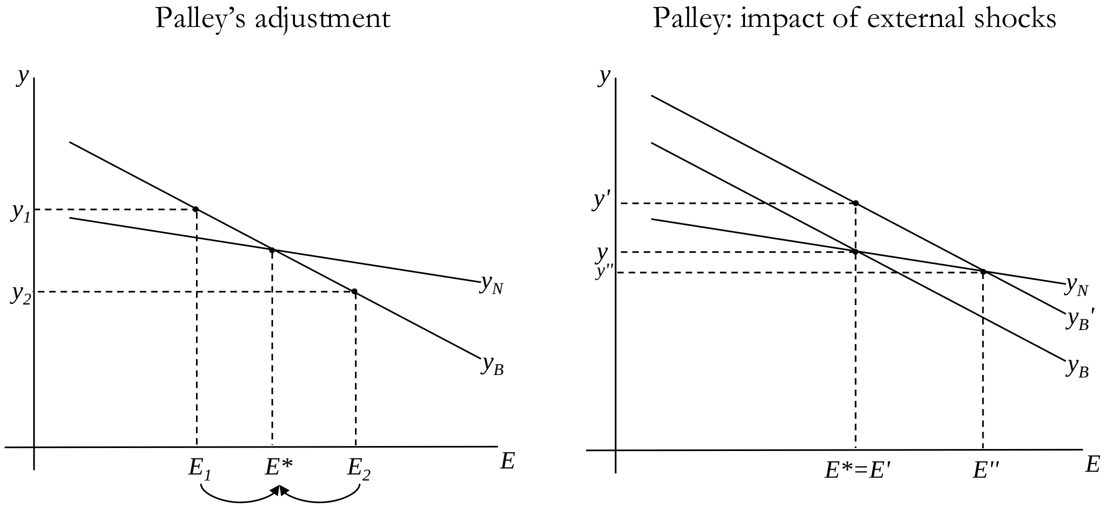
As we assume that the BPCG rate determines the actual rate of growth, $\frac{\partial y}{\partial E} < 0$, and thus y_N is negatively related to the degree of capacity utilization[5]. However, the slope of this curve is lower than the slope of y_B because it is multiplied by v, which is lower than the unity. Hence,

[4] Some studies, such as León-Ledesma & Lanzafame (2010), and Lanzafame (2014), have investigated the relationship between the BPCG and the natural growth rates, and found unidirectional causality from the BPCG rate to the natural growth rate.

[5] The use of the term capacity utilization we employ here is related to utilization of factors of production, so it could be either related to capital utilization or labour utilization (employment) depending on the type of constraint faced by the economy (capital or labour).

considering a general representation of the functional form $\pi = \pi(E)$, the following graph represents Palley's adjustment mechanism:

Figure 2.1 – Adjustment and external shock in Palley's adjustment



In Palley's adjustment, the growth rate of the economy is, in the end, determined by the natural rate of growth. If the BPCG rate is higher than the natural growth rate, such as in y_1, the economy will grow faster than this potential and the degree of capacity utilization will increase towards E^*, reducing the actual rate of growth faster than the natural rate of growth until their convergence. On the other hand, if the actual rate is lower than the natural rate of growth, such as in y_1, the degree of capacity utilization will decrease towards E^*, and the adjustment will result in higher growth rates. As a conclusion, the actual rate of growth will adjust towards the natural rate of growth.

An increase in the BPCG rate, due to a positive external shock in z, may have an ambiguous effect in the economy. As a result of a foreign demand shock the BPCG rate increases and becomes higher than the natural rate of growth, such as in y'. However, because the actual rate of growth is higher than the natural rate of growth, the degree of capacity utilization will increase, and the economy will reach a rate of growth lower than the original ($y'' < y$). Thereby, in Palley's scenario, a foreign demand shock has little impacts on the long run equilibrium growth rate. This characterises a *quasi-supply-determined growth*, adjusting through the degree of capacity utilization.

Palley's adjustment is in a way similar to Krugman's (1989) approach to the relationship between total factor productivity and income elasticities. Krugman argues that the 45-degree rule (which nothing else than Thirlwall's law)[6] is explained by the growth of total factor productivity,

[6] Krugman (1989) refers to the 45-degree rule as the same economic growth stylized fact in which the Post-Keynesian literature used to address the balance of payments constrained growth rate (Thirlwall's Law). Both are identical. For Krugman, nevertheless, the demand elasticities for export and imports depends on the rate of

which is strictly related to the specialization in trade. In his perspective, fast growing countries increase their share of world markets expanding the range of goods that they produce. This mechanism results in favourable income elasticities. Thereby, it is the increasing returns of specialization which produces higher elasticities and faster growth, which characterises a supply-side explanation for this rule.

Therefore, in Palley and Krugman's views the income elasticities are endogenous to the productivity growth. In spite of having different approaches (in Palley's view it is due to degree of capacity utilisation, and for Krugman it is due to trade specialization) for both, the growth rates in the long run are (quasi-)supply determined.

2.2. Setterfield's alternative approach

In contrast to Palley's view, Setterfield (2006) provides an alternative solution. The author argues that productivity growth is a positive function of the degree of utilisation, and thus the natural growth rate may be endogenous to actual growth rate.

Assuming the same supply and demand growth rates (equations (2.1) and (2.2)), Setterfield suggests a different process of adjustment capable to reconcile both growth rates. According to him, although Palley's adjustment may take place, the sensibility of productivity to the actual rate of growth (the Verdoorn's coefficient) is endogenous to the rate of capacity utilization.

The rationale behind Setterfield (2006)'s argument is that the extent to which any rate of output growth will induce productivity growth (i.e., the precise size of Verdoorn's coefficient) is a direct function of the Verdoorn's coefficient. If the level of demand is low relative to the full capacity utilization, then firms will be less likely to engage in innovation, technical change and organizational change from which productivity gains materialize. In other words, "more productivity growth is induced by a goods market that is both tight and rapidly expanding" (Setterfield, 2006, p.54). Thereby, we have that

$$v = v(E), v' > 0 \tag{2.7}$$

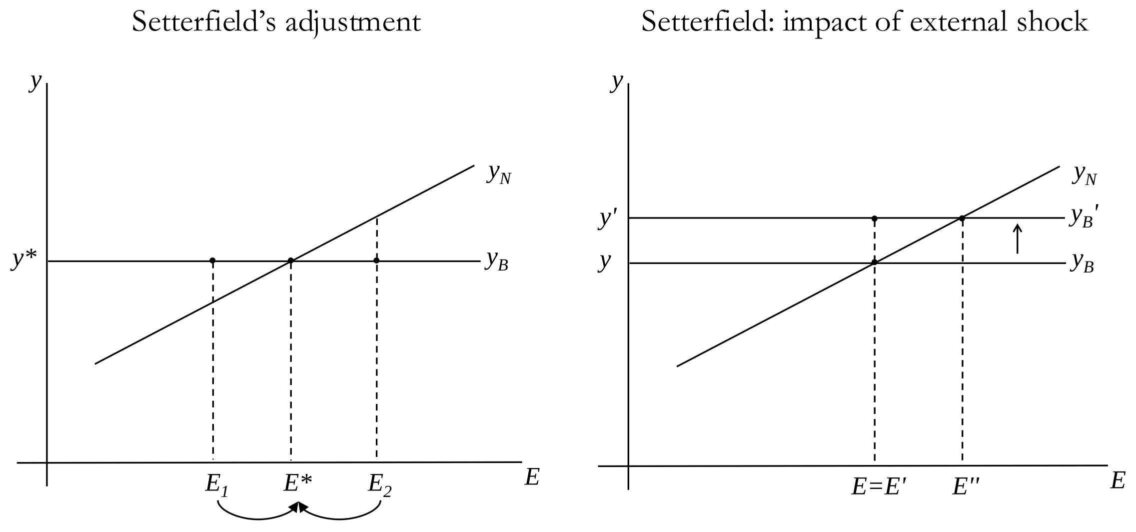
If the demand growth rate is higher than supply growth rate, the increase on the rate of capacity utilization will increase the Verdoorn's coefficient and thus the natural growth rate. As a result, the economy once again achieves a sustainable steady state growth equilibrium, which is demand determined. Essentially, the author considers, based on the natural rate of growth in (2.1), that:

$$\frac{\partial y_N}{\partial E} = \frac{\partial y_N}{\partial v}\frac{\partial v}{\partial E} = y\frac{\partial v}{\partial E} > 0 \tag{2.8}$$

technical change. Productivity and technology are the factors behind the long run growth, and demand plays no function.

In which $\frac{\partial v}{\partial E} > 0$, then a higher degree of capacity utilization increases the natural rate of growth, and thus Palley's adjustment does not need to take place in order to the natural and actual growth rates be the same. That is to say, in Setterfield's case, the supply growth curve is upward slopping and the demand growth curve is perfectly elastic. A stylized representation can be observed below:

Figure 2.2 – Adjustment and external shock in Setterfield's version



In Figure 2.2, when the demand growth rate (or the BPCG rate) is higher than the supply growth rate, capacity utilization increases, thereby raising the Verdoorn coefficient and, consequently, fully accommodating the supply side on the demand side. Conversely, if the supply growth rate is greater than the growth of the aggregate demand, then the wane in the capacity utilization brings down the Verdoorn coefficient, and hence the supply side of the economy accommodates the development of the demand side.

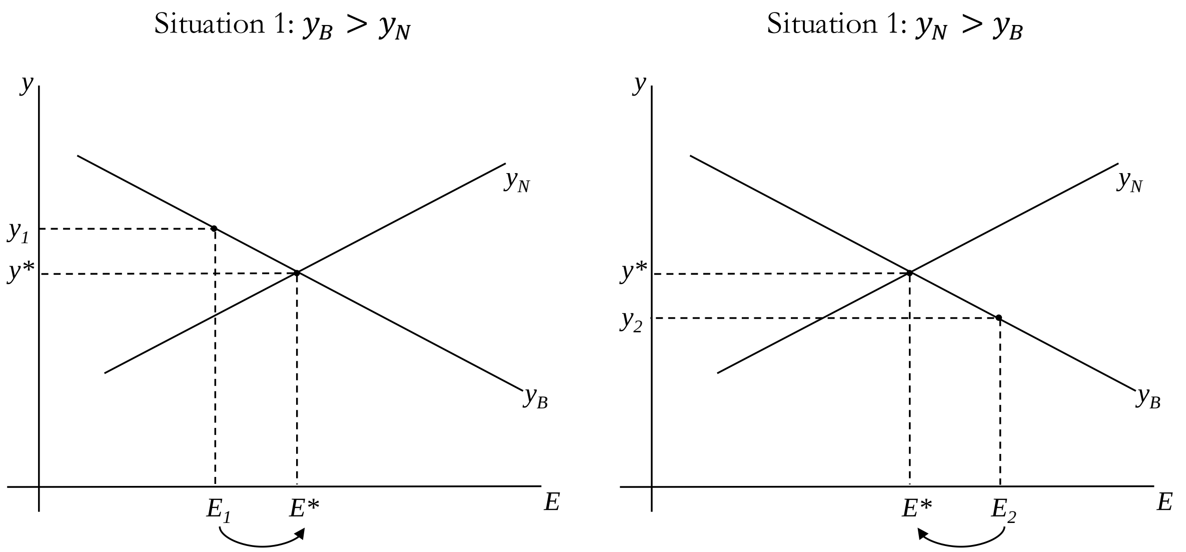
In this scenario, a positive shock in world demand has two effects on the economy. First, the BPCG rate increases and becomes higher than the natural rate of growth, such as in y'. However, if the actual rate of growth is higher than the natural rate of growth, degree of capacity utilization rises until the point where the degree of capacity utilization is E''. Hence, the economy returns to the equilibrium in a situation in which it grows faster than before, characterising a *fully-demand-determined growth*. In Setterfield's adjustment, a positive shock affect both the equilibrium growth rate in the long run and the degree of capacity utilization.

2.3. Adjustments in Palley-Setterfield approaches

Palley and Setterfield developed two alternative, but not mutually exclusive, approaches for the process of convergence between supply and demand. By assuming the endogeneity of the income elasticity of imports to capacity utilization, Palley states that the BPCG rate converges towards the natural growth rate. Setterfield's assumption, on the other hand, argues that only the Verdoorn's coefficient is endogenous to the capacity utilization, yields a binding demand constraint on long-term growth.

Hence, we can describe two situations: (1) the actual rate of growth is higher than the natural rate of growth, and (2) the natural rate of growth is higher than the actual rate of growth:

Figure 2.3 – Supply and demand adjustment process



In both situations, if supply and demand are growing at different rates, the degree of capacity utilization E is different from E^*, which is an unstable and unsustainable situation.

In the first situation, the BPCG rate is higher that the natural rate of growth. As we can see from Figure 2.3, the degree of capacity utilization is lower than its stable equilibrium (E^*). The degree of capital utilization then increases, once the actual rate of growth is higher than the natural rate of growth. As a result, the BPCG rate will decrease and adjusts towards the natural rate of growth and the natural rate of growth will increase and adjusts towards the BPCG rate. Thereby, E^* is not only a stable point, but also an attractor for both growth rates.

In the second situation we have exactly the reverse. If the BPCG rate is lower than the natural growth rate, then the degree of capital utilization is higher than the stable point, which implies that the degree of capital utilization will decrease. In this situation, the BPCG rate will increase and adjusts towards the natural growth rate at the same time as the natural growth rate will decrease and adjusts towards the BPCG rate. Thereby E^* is again an attractor for both growth rates.

This formulation offers the basis to describe to what extent the supply and demand are endogenous to the degree of capacity utilization. Based on Palley and Setterfield approaches, the degree of capacity utilization changes in the long run to adjust natural and actual rate of growth.

2.4. McCombie's critique to the Palley-Setterfield adjustment

According to McCombie (2011), Palley's (2003) adjustment does not occur. Short-run income elasticities may change due to short-run cyclical effects. However, in the long-run, income elasticities are far more difficult to change. The author argues that "the increase in output growth would potentially increase the growth of imports, but this is offset by the increased capacity that allows all the increased demand to be met by domestic production and the long-run income elasticity of demand for imports falls commensurately"(McCombie, 2011, p. 373). In short, the increase of imports will raise the potential output and thus the degree of capacity utilization will return to its original level. Consequently, the long-run income elasticities of demand for imports will not change.

The author also criticises Setterfield's (2006) approach because he does not consider that the growth rate of labour force and technical change are endogenous to the rate of capacity utilization, and so there is no unique rate of growth associated with a stable rate of unemployment. Due to that, the growth rate is balance-of-payment constrained, and demand-determined even if the Verdoorn's coefficient is not endogenous. There is no need for changes in the degree of capacity utilization in order of the economic growth to be demand-driven.

Palley and Setterfield analysis relies on the idea that short-run cyclical effects may affect, respectively, the income elasticities and the Verdoorn's coefficients. In these approaches, capacity utilization adjusts supply and demand growth rates. However, if one considers that there is a natural rate of capacity utilization, E^*, which the economy tends to fluctuate around, their adjustments become implausible. Thus, the long-run income elasticities and the Verdoorn's coefficient are far more difficult to change, which may not take place in a growing and stable economy.

McCombie (2011) argues that it is necessary to consider that the labour force and technical change are endogenous to output growth:

$$l'(E) > 0, \lambda'(E) > 0 \tag{2.9}$$

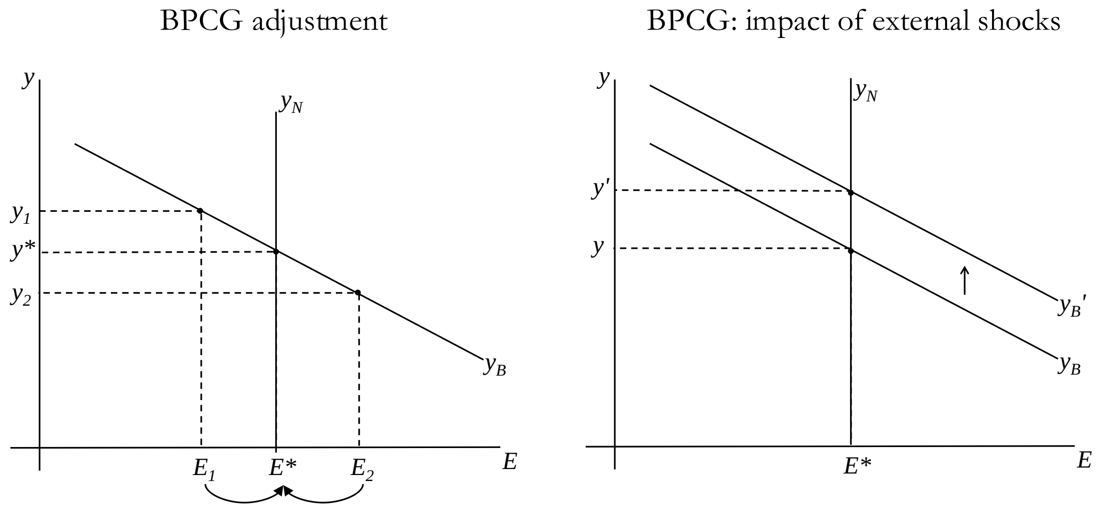
Although it does not change the relationship between the BPCG rate and the degree of capacity utilization, it means that $\frac{\partial y_N}{\partial E}$ is even higher: the natural rate of growth is more elastic in relation to the degree of capacity utilization than it was before considering (3.1). Based on (2.1), (2.6) and (3.1) we have that

$$\frac{\partial y_N}{\partial E} = \frac{\partial y_N}{\partial v}\frac{\partial v}{\partial E} + \frac{\partial y_N}{\partial y}\frac{\partial y}{\partial E} + \frac{\partial y_N}{\partial l}\frac{\partial l}{\partial E} + \frac{\partial y_N}{\partial \lambda}\frac{\partial \lambda}{\partial E} = \underbrace{y\frac{dv}{dE}}_{(+)} + \underbrace{v\frac{dy}{dE}}_{(-)} + \underbrace{\frac{dl}{dE}}_{(+)} + \underbrace{\frac{d\lambda}{dE}}_{(+)} \tag{2.10}$$

This equation shows that there are many factors affecting the relation between the natural rate of growth and the degree of capacity utilization. However, most of them are positive, which means that the natural rate of growth tends to be positively related to the degree of capacity utilization.

Based on Cornwall (1977), McCombie (2011) argues that even mature economies have an elastic labour force and the technical progress is stimulated by the increase of degree of capacity utilization due to a great number of factors, such as an increase of R&D expenses and investments in more productive capital. Thereby, he argues that countries are not usually supply constrained, but balance-of-payment constrained. Based on the analytical tool developed in the last section, we can consider that what the author is saying is that the natural growth rate curve is vertical, or, in his words that "there is no unique rate of growth associated with a constant rate of unemployment". Because of the elasticity of l and λ, we represent a situation in which the effects of E on y have infinite elasticity. In this sense, the natural rate of growth is, in the limit, vertical. The following figure presents what we define as the BPCG scenario:

Figure 2.4 – Adjustment and external shock in BPCG scenario



The interesting point about this case (which McCombie considers as the most common case) is that growth rates are determined only by the demand side, once there are multiples natural growth rates associated with a unique degree of capacity utilization. The BPCG rate determines the actual growth rate in the long run and the natural growth of rate adjusts towards this rate through labour supply adjustments (i.e. Migration between sectors of a dual economy, or international migration of workers). If a country starts growing faster without experiencing a structural change on its BPCG rate, the economy will tend to E^* and, consequently, to the original BPCG rate.

As can be seen from the first graph, E_1 and E_2 are not equilibrium points. If the economy is on the left side of E^*, the actual rate is lower than the natural growth rate and the degree of capacity utilization will increase. On the other hand, if the economy is on the right side, the actual growth rate will be higher than the natural growth rate, and the degree of capacity utilization will decrease. Thereby, E^* is an attractor which forces the economy to be always balance-of-payment constrained.

Such as in Setterfield's (2006) scenario, in order to achieve a higher growth rate, a country has to change the income elasticities, reducing the balance-of-payment constraints. If the economy is not able to do so, the growth rate will increase given the natural degree of capital utilization only due to a faster growth of z. As shown in the graph on the right, a positive external shock promotes an increase on BPCG rate, and the curve y_B shifts towards a faster growth rate.

This case gives an explanation on why an economy can be balance-of-payment constrained even considering the supply side. If labour force and technological change are endogenous to the degree of capacity utilization, the supply side is not a constraint for growth (it is completely accommodated by demand shocks), and thus the economy is demand-driven. Furthermore, different from the Setterfield's (2006) scenario, the degree of capacity utilization does not change – which is coherent with the hypothesis that there is a natural rate of capacity utilization to which the economy tends to fluctuate around. Thereby, in McCombie's (2011) view, the BPCG scenario is the only one able to provide a plausible adjustment once it does not rely on changes in the degree of capacity utilization in the long run.

3. Capital and labour constraints: necessary conditions for reconciliation

3.1. Labour and capital constraints in McCombie's critique

Based on McCombie's critique for the need of a reconciliation between supply and demand growth rates, Setterfield (2013) proposes an explicit account of the supply side compatible with a Kaldorian macro-dynamics. The author assesses the conditions in which McCombie's (2011) critique is valid, and hence he re-affirms the need to seek mechanisms capable to reconcile these two growth rates. According to Setterfield (2013), McCombie (2001) criticises the Palley-Setterfield approach (Palley, 2003; Setterfield, 2006) based on the idea that actual rate of growth is always bellow its potential, and thus it is not constrained neither by capital nor by labour. In this sense, McCombie's (2011) critique is valid only under very specific conditions.

Setterfield (2013) uses an explicit description of the supply side. Potential growth rate (rather than natural growth rate) is given by a Leontief function[7]:

$$Y_P = \min\left[\frac{L_c}{a}, \frac{K_c}{b}\right] \tag{3.1}$$

where Y_P is the potential growth rate, L_c is the labour available, K_c is the capital available, a is the potential labour output ratio, and b is the potential capital-output ratio.

Two possible constraints may emerge from the Leontief equation. First, a labour constraint exists if the actual rate of grow is higher than the growth rate of $\frac{L_c}{a}$. Second, a capital constraint emerges if the economy grows faster than $\frac{K_c}{b}$. From this moment in our analysis we stop calling both capital and labour constraints as capacity utilization. We specify when addressing each of them.

3.1.1. Labour constrained economy

From the first part of Leontief function in (3.1), it is possible to describe the origin of possible labour constraints. In this case, the dynamics of potential output is given by:

$$Y_P = \frac{L_c}{a} \rightarrow y_P = n - \hat{a} \tag{3.2}$$

From this equation, it is clear that there are two channels which actual growth rates affect y_P. First, the Verdoorn's law may affect the growth of labour-output rate, which the inverse of labour productivity, as follow:

$$-\hat{a} \equiv q = \lambda + \nu\, y \tag{3.3}$$

In which $\hat{a}$ is the growth of labour-output ratio. Second, the labour force may be endogenous to the output growth, such as argued by McCombie (2011):

$$n = \gamma + \delta y \tag{3.4}$$

γ is the exogenous growth of labour, and δ is the labour-elasticity to output.

[7] See Setterfield (2013) for the arguments in favour of adopting a Leontief function to describe the supply side.

Hence, growth rate of potential output can be written as a linear function of exogenous technical change and labour force growth, and endogenous technical change and labour force growth:

$$y_P = \gamma + \lambda + (\delta + \nu)y \tag{3.5}$$

Thereby, the impact of an increase of the actual rate of growth (which is given by the demand side) impact the labour side of potential growth rate as follow.

$$\frac{d(y_P)}{d(y)} = \delta + \nu \tag{3.6}$$

Based on this relationship, Setterfield (2013) concludes that there is only one specific case in which the economy will not face a labour constraint: when $\delta + \nu = 1$. In this case, we have that y_P and y will grow at the same rate not only in the long run, but, most importantly, also in the short run. Thereby, the economy will not present variations in the degree of capacity utilization, and hence there is no need for reconciliation. However, it may not be the case. If $\delta + \nu < 1$, the economy will face labour constraints, and a reconciliation of supply and demand is needed.

3.1.2. Capital constrained economy

The constraint presented above only takes into account the labour side of the Leontief function. Setterfield (2013) also presents the necessary conditions for having a capital constraint in the economy. Such as before, the dynamics of potential output can be expressed as follow (observing from the capital side):

$$Y_P = \frac{K_c}{b} \rightarrow y_P = \widehat{K_c} - \hat{b} \tag{3.7}$$

He assumes that, according to Kaldor (1961), b is constant in the long run. Consequently, there is only one possible response for a faster growth in y_P, which is a faster growth of capital accumulation. Hence, potential output growth rate can be described as

$$y_P = \widehat{K_c} \tag{3.8}$$

Setterfield argue that the assumption that $\widehat{K_c}$ is independent of the growth process is distinctly un-Kaldorian. He then describes investment based on a simple accelerator mechanism, as follows:

$$\Delta K_c = I = b\,\Delta Y = byY \tag{3.9}$$

Equation (3.9) is essential to understand the origin of capital constraints. First, the author assumes that there is no depreciation, and thus growth of capital is equal to investment. Moreover, given a constant capital-output ratio, Setterfield assumes that investment is determined uniquely by the growth of output.

Finally, we have that:

$$b = \frac{K_u}{Y} = \frac{K_c}{Y_P} \tag{3.10}$$

And

$$u = \frac{Y}{Y_P} = \frac{K_u}{K_c} \tag{3.11}$$

In which u is the degree of capital capacity utilization, and K_u is the capital employed. We can write the rate of growth of potential product as:

$$y_P = \widehat{K_c} = \frac{\Delta K_c}{K_c} = b\,y\frac{Y}{K_c} = \frac{K_u}{Y}y\frac{Y}{K_c} = u\,y \tag{3.12}$$

Finally, analogous to the analysis of labour constraints, we can describe the impact of a faster growth of the actual rate of growth on the growth rate of potential output, in order to assess the necessary condition for achieving a stable growth:

$$\frac{d(y_P)}{d(y)} = u \Rightarrow d(y_P) = u\,d(y) \tag{3.13}$$

Equation (3.13) shows that the only way potential and actual outputs grow at the same rate is when $u = 1$. However, as Setterfield argues, this case is very specific and data shows that, although the degree of capacity utilization is very volatile, in the US it fluctuates around 0.80 in the last three decades. Hence, in the context of a long-run model, this assumption is very unrealistic.

Thereby, based on capital and labour constraints, the demand side is fully accommodated by the supply side, (à la McCombie (2011)) only under the very specific case where $u = 1$ and $(\delta + v) = 1$ occurs. Consequently, according to Setterfield (2013), the need for a reconciliation of supply and demand based on Palley-Setterfield mechanisms re-emerges.

3.2. Capital constraints in monopolistic economies

Setterfield (2013) presents two approaches to argue for the need of reconciliation: one based on capital constraints and, the other, on labour constraints. His approach on the capital constraints, however, raises some concerns on the assumption behind the use of only an accelerator mechanism in the investment equation. The problem rises from the static characteristics of this accelerator formulation, once investments respond only to changes in demand, ignoring the degree of capacity utilization. When considering $I = b\,\Delta Y$ it implies that capitalists invest only to increase the actual output according to the demand growth. However, in a monopolistic economy, capitalists also aim to keep the degree of capital utilization unchanged[8]. Essentially, by doing this, the growth rate of Y_P will be equal to the growth rate of demand, and hence there will be no capital constraints.

Based on the same assumption made by Setterfield (2013) that b is constant (Kaldor, 1961) and that there is no depreciation, we have that

$$I = \Delta K_c = b\,\Delta Y_P \tag{3.14}$$

Once $u = \frac{Y}{Y_P}$, we can write investment in terms of capacity utilization:

$$I = b\left(Y_P - Y_{P_{-1}}\right) = b\left(\frac{Y}{u} - \frac{Y_{-1}}{u_{-1}}\right) \tag{3.15}$$

From this equation it is possible to verify how investment can be a function of output, as stressed by Setterfield (2013), but also of the degree of capacity utilization. By assuming a monopolistic economy, where capitalists invest to keep the degree of capacity utilization unchanged, the investment function can be written as[9]:

[8] Empirical evidences, as presented in Caiani et al. (2016), show that firms aim for normal rates of utilization. Moreover, excess capacity works as an entry barrier strategy against new firms. Lavoie (2014) offers a survey on the topic.

[9] It does not mean that capacity utilization keeps unchanged. The assumption is that investment is made trying to keep it unchanged. However, it may vary due to many factors, including a faster demand growth or investors' difficulties to find funding for their investment.

$$\Delta K_c = I = b \frac{\Delta Y}{u} = \frac{b}{u} yY \tag{3.16}$$

This equation is very similar to Setterfield's (2013) accelerator mechanism. The difference between (3.16) and (3.11), is that whilst in (3.16) the degree of capacity utilization keeps unchanged, in Setterfield's approach it changes over time (although not explicitly). From this equation for the accelerator, the growth of potential output is given by:

$$y_P = \widehat{K_c} = \frac{\Delta K_c}{K_c} = \frac{v}{u} y \frac{Y}{K_c} = \frac{K_u yY}{uYK_c} = y \tag{3.17}$$

which means that $d(y_P) = d(y)$. Thereby, we can argue that if investment is oriented to keep the degree of capacity utilization unchanged, <u>there is no capital constraint</u>[10]. The previous result (the one of Setterfield (2013)) was reached because of the sole static accelerator mechanism. Once it is assumed that investment is a function both of demand growth and the degree of capacity utilization, the demand side will be fully accommodated by the supply side even if $u < 1$.

4. General model: a reconciliation

Although capital constraints do not emerge if investment is oriented to keep the degree of capacity utilization unchanged, as presented in the end of the last section, labour constraints may emerge if the summation of labour supply elasticity to output and the Verdoorn coefficient is lower than the unity. The difference between capital and labour constraints is that whilst in a monopolistic economy capital supply is *fully*-endogenous to demand growth, and, consequently, in the absence of funding constraints, all demand for capital is fulfilled by its supply, in the case of labour, supply may have its own dynamics even in the long run. Thereby, a reconciliation of supply and demand is needed in this case.

With the aim of designing a general model capable of encompassing Palley's (2003), Setterfield's (2006) and McCombie's (2011) approaches, as well as the debates on Setterfield (2013), we initially raise the convergent and divergent aspects all these different perspectives. First of all, all approaches present a long-term demand function, which is given by the BPCG model (Thirlwall's law), and a long-term supply function, which is given by the natural rate of growth. We can observe that McCombie's (2011) critique is not about the hypothesis that income-elasticity of demand and Verdoorn's coefficient responds to the rate of capacity utilization. His critique encompasses how Palley (2003) and Setterfield (2006) do not address other factors that respond to actual output growth. In a conciliation proposal to this critique, we can write the long-term demand growth, y_B, and the long-term supply growth, y_N, as follow:

[10] We are not neglecting here that capital constraints will never emerge. Countries can have funding problems both domestically and internationally. However, these capital constraints do not emerge from Setterfield's (2013) critique if firms invest to maintain a constant level of capital utilization.

$$y_B = \frac{\varepsilon}{\pi_0+\pi_1 E} z \tag{4.1}$$

$$y_N = n + \lambda + (v_0 + v_1 E)y \tag{4.2}$$

where ε is the income-elasticity of demand for exports, π_0 is the exogenous component of the income-elasticity of demand for imports, π_1 is the sensitivity of the income-elasticity of demand for imports to the capacity utilization (Palley-effect), E is the rate of capacity utilization, n is the labour-force supply growth rate, λ is the exogenous technological change, v_0 is the exogenous component of the Verdoorn coefficient, v_1 is the sensitivity of the Verdoorn coefficient to the capacity utilization (Setterfield effect), and y is the actual growth rate. In this formulation we implemented linear representations for $\pi = \pi(E)$ and $v = v(E)$ for the sake of simplification.

Before introducing McCombie's (2011) critiques to these models, we need to define the rate of capacity utilization. Since we are dealing only with labour constraints, we can define it as the degree of labour utilization or simply employment rate (E). This rate provides the dynamics of the model. Without loss of generality,[11] we explore the labour market dynamics, as E is given by the ratio of effectively absorbed labour (L) and the supply of labour (N).

$$E = \frac{L}{N} \tag{4.3}$$

or, in terms of growth rates:

$$e = l - n \tag{4.3b}$$

in which the lower cases mean growth rates.

Equation (4.3b) is the main equation for the dynamic adjustment of supply and demand. Rather than using the approach presented in the systematization of the Palley-Setterfield-McCombie controversy in Section 2, which is based on the relation between E and y, here we analyse the dynamics of e. It will be the lack of adjustment on supply of labour to the labour force effectively employed that will explain the dynamic adjustment of supply and demand.

4.1. The Palley-Setterfield adjustment revisited

[11] As discussed in the early section, the only constraint that needs conciliation is the labour constraint. Capital constraint may be important due to the lack of funding, but it does not emerge from the dynamics of supply and demand if firms invest to keep capacity utilization unchanged.

In our formulation, we derive the growth rate of employment rate from explicit equations for the growth of labour supply, n, and the growth of labour force effectively employed, l. The labour market needs to adjust in order to reach a stable dynamics, otherwise there will be no convergence between supply and demand growth rates.

For both Palley (2003) and Setterfield (2006), the supply of labour is not sensitive to the rate of capacity utilization. There are many reasons for that to be the case in an advanced economy, which is not dual *à la* Lewis (1954). In developing economies, agricultural and traditional services sectors act as a reservoir of surplus labour for more productive sectors, and hence these advanced sectors face a perfectly supply labour force schedule (McCombie and Thirlwall, 1994). However, as countries reach an advanced stage of development, this surplus labour is exhausted, and the supply of labour becomes perfectly inelastic. Following this idea, we consider that n is constant, and given by:

$$n = n_0 \tag{4.4}$$

n which n_0 is the exogenous (and unique) component of the labour supply.

To examine the dynamics of the effectively employment of labour, l, we consider it is positively associated to the actual growth rate, and negatively associated to the natural growth rate. The higher is the actual growth rate, the higher is the growth rate of labour effectively employed. However, because productivity is also endogenous to output growth, the natural rate of growth can reduce the labour demand, and hence it affects negatively the growth rate of actual employed labour. Thereby, the growth rate of the labour force effectively employed may be given by:

$$l = \phi(y - y_N) + n \tag{4.5}$$

in which ϕ is the elasticity to the adjustment mechanism.

As McCombie (2011) highlights, the identity $p = y - l$, where p is the productivity growth, must be valid. It means that either actual growth rate is given by productivity growth or by employment growth. In order to have this identity, ϕ must be equal to one.[12] Thereby, replacing equation (4.2) in (4.5a), and considering that $\phi = 1$, we have:

$$l = -\lambda + (1 - v_0 - v_1 E)y \tag{4.5b}$$

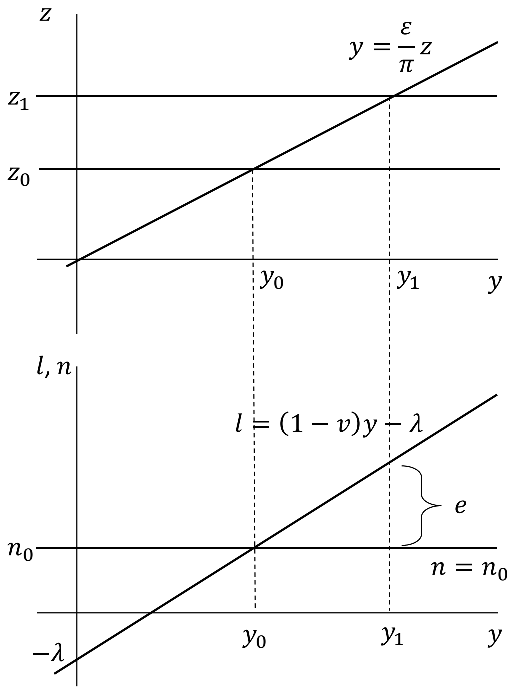
[12] Replacing $p = y - l$ in the natural rate of growth we have that $y - y_N = \phi(y - y_N)$. This equation has two possible solutions: $y = y_N$ and $\phi = 1$. If y is not necessarily equals to y_N, as we are supposing, ϕ must be equal to one.

Equation (4.4) and (4.5b) provides the central adjustment mechanism of the model, since they determine the growth of employment rate, e. On the one hand, this growth rate is positively affected by the growth rate of labour effectively employed, which is given by equation (4.5b), and, on the other hand, the higher is the growth rate of labour supply, which is given by equation (4.4), the lower is the growth of employment rate.

Until now, there were no link between actual output growth, y, and the long-term demand growth, which is given by the BPCG growth rate, y_B. If we assume that the long-term demand growth provides a good approximation for the actual output growth, $y_B = y$ (see footnote 4), then equations (4.1), (4.2), (4.3b), (4.4) and (4.5b) are enough to solve the Palley-Setterfield version of the model.

Figure 4.1 presents two graphs: the upper one shows the BPCG, where, given z and the elasticities ratio, one can determine y. The lower one shows the relationship between l, n, e and y. Both graphs assume E as given, so they are showing all schedules in the short run. In this case, the supply of labour, n, is perfectly inelastic to y, and hence it is a horizontal line. Variations in the actual growth rate do not affect the labour supply since, as discussed before, for Palley (2003) and Setterfield (2006), it is exogenously given. Labour effectively employed, on the other hand, is positively related to output growth if the composed Verdoorn coefficient is lower than one. If the world output growth, z, is the one that provides y_0 (given the elasticities ratio), the economy is in equilibrium, since labour effectively employed is equal to labour supply, and hence $e = 0$ (there is no variation in E). In this case, both natural and actual growth rates are equal, and there is no need for an adjustment.

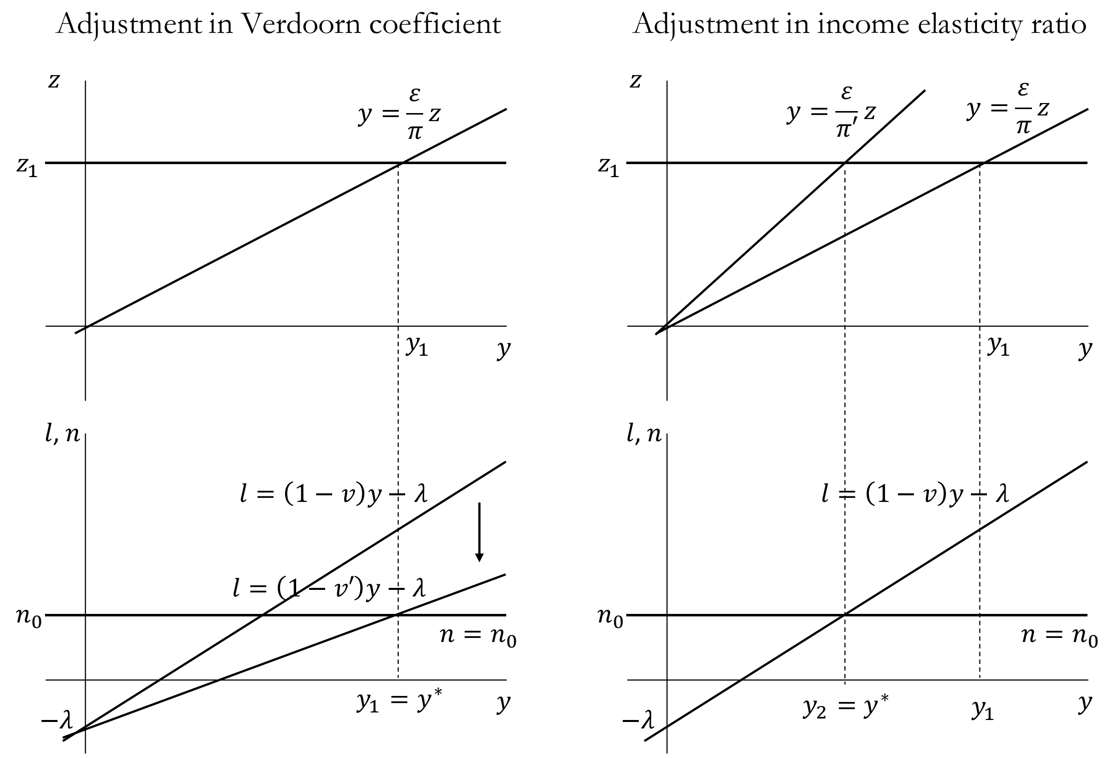
Figure 4.1 – Labour supply and effectively employed in the Palley-Setterfield case (short run)



An adjustment, however, is necessary if the world output growth is not exactly equals to z_0. If, for example, it is higher than that (z_1), the actual growth rate will be higher, and hence the growth of labour force effectively employed will be higher than the growth of labour supply. Since there is nothing that guarantee that $-\lambda + (1 - v)y = n_0$, natural and actual growth rates may be different. In this case, employment rate, E, is not constant since there is a gap between labour effectively employed and labour supply. In the case of y_1, e is positive, since the growth rate of employment is higher than the growth rate of labour supply.

The fact that e is different from zero implies that employment rate, E, is changing, and hence the either the Verdoorn coefficient or the income-elasticity of demand for imports (or both) is also not constant. As can be seen from Figure 4.2, these adjustments can be understood from either change in the labour supply schedule or change in the actual growth rate, y.

Figure 4.2 – Adjustment of labour supply and effectively employed in the Palley-Setterfield case



These two extreme cases are those proposed originally by Setterfield (2006) and Palley (2003), respectively. In the first of these adjustments, if $e > 0$, the employment rate will increase, and thus the Verdoorn coefficient will also increases (from v to v'). The schedule of the labour effectively employed labour growth will rotate clockwise, and a new equilibrium will emerge, where the actual growth rate is higher. In this case, demand fully accommodates supply, so the growth rate of an economy is *fully-demand determined*.

In the second case, if $e > 0$, the employment rate will increase, but instead of an adjustment in Verdoorn coefficient, there will be an adjustment in the income-elasticity of demand for imports (from π to π'). Here, there is no change in the labour growth schedules. Instead, the actual growth rate will reduce towards an equilibrium (the elasticities ratio schedule will move anti-clockwise). In this case, supply fully accommodates demand, and hence growth rate of an economy is *fully-supply determined*.

Before analysing its dynamics, however, it is important to evaluate the necessary conditions for the stability. Replacing (4.4) and (4.5b) in (4.3b), and considering that $y_B = y$, we have:

$$e = (1 - v_0 - v_1 E)\frac{\varepsilon}{\pi_0 + \pi_1 E} - (\lambda + n_0) \tag{4.6}$$

For the model to be stable, E cannot be explosive. Thereby we must have:

$$\frac{de}{dE}=\frac{-v_1(\pi_0+\pi_1 E)-\pi_1(1-v_0-v_1 E)}{(\pi_0+\pi_1 E)^2(1+\theta)}\leq 0 \tag{4.7}$$

Once the income elasticity of demand for imports $\pi=\pi_0+\pi_1 E$ is expected to be positive, and the Verdoorn coefficient $v=v_0+v_1 E$ is expected to be lower than one, the stability conditions are $v_1\geq 0$ and $\pi_1\geq 0$.

Finally, once e is stable, we can consider that in the long run, $e=0$. Thereby, it is possible to find the equilibrium of the rate of capacity utilization using equation (4.6). Making all necessary reallocation we have the steady state for employment:

$$E^*=\frac{(1-v_0)\varepsilon z-\pi_0(\lambda+n_0)}{v_1\varepsilon z+\pi_1(\lambda+n_0)} \tag{4.8}$$

4.2. The McCombie adjustment revisited

According to McCombie (2011), the Palley-Setterfield (Palley, 2003; Setterfield, 2006) adjustment ignores that both labour supply and technological progress are endogenous to the rate of capacity utilization, and hence to the actual output growth. That is a central aspect of labour and technology dynamics. Cornwall (1977) argue that even in advanced economies the supply of labour may be elastic to wage and output growth. Although Lewis' view on labour surplus is concerned with less advanced countries, Cornwall (1977:95) argue that "employment patterns were demand determined in the various market economies in the post-war period", and "when entrepreneurs in the manufacturing sectors of different economies wanted labour they found it one way or another". Thereby one cannot ignore that supply of labour is endogenous to output growth, as in Palley-Setterfield approach.

Moreover, McCombie (2011) argue that technical progress is stimulated by the increase of degree of capacity utilization due to a great number of factors, such as an increase of R&D expenses and investments in more productive capital. However, as the Verdoorn coefficient is already considering the impacts output growth on technological change,[13] for simplicity we will consider here only the impact of actual output growth on labour supply.

Let us assume that supply of labour responds to output growth to guarantee that the natural rate of growth will not differ even in the short-term, such as presented in Figure 2.4 above. This assumption implies that $y_N=y$, and that the growth rate of labour supply will be

[13] It is possible to consider it more precisely by including a term in the productivity that accounts for deviation from capacity utilization, $p=\lambda+vy+c(y-y_N)$. It is important to avoid that rather than measuring the Verdoorn coefficient, we could be measuring Okun's law (Magacho & McCombie, 2017). However, for simplicity we will ignore it here.

the adjustment variable. Thereby, we must re-define it, as it is not exogenously given, as before. Considering again that the identity $p = y - l$ is valid, and that the productivity is given by Verdoorn's law, $p = \lambda + vy$. Replacing equation (4.1) in this identity and considering that $y_N = y$, we have that:

$$n = l \tag{4.9}$$

From this equation, labour supply adjusts completely to the labour demand to guarantee that there will be always convergence between the natural rate of growth and the actual output growth. In terms of the diagram presented above, what we have is a labour supply growth schedule that is coincident with the labour effectively employed labour growth schedule. It implies that there is no gap between labour supply and employment, and hence there is no change in the employment rate.

Once the natural rate of growth is defined by (4.1), and the actual output growth is defined by the BPCG rate, which is given by (4.2), we have always that $y_B = y = y_N$. Labour supply is thus given by

$$n = (1 - v)\frac{\varepsilon}{\pi} - \lambda \tag{4.10}$$

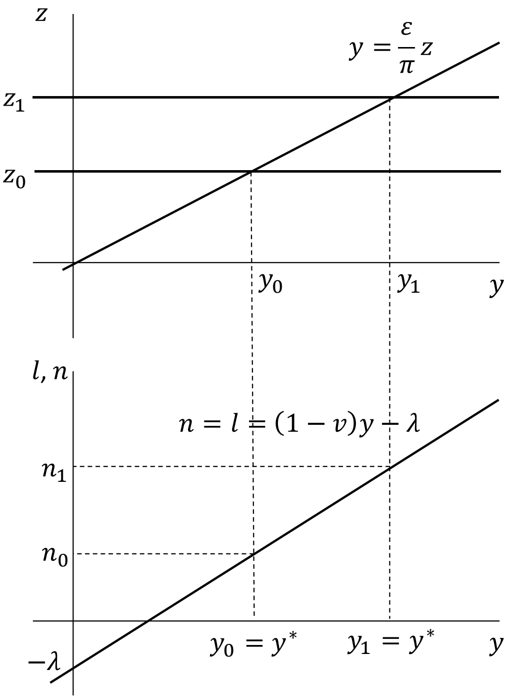
where π and v are constant since e is always equal to zero, and hence $E = E^*$.

Since E is fixed, there is no need to analyse the stability. However, we can calculate E^*. In order to do this, we can recall equation (4.6), but rather than considering $n = n_0$, we assume that equation (4.9) is valid. Thereby, because $e = 0$, we have:

$$E^* = \frac{(1 - v_0)\varepsilon - \pi_0(\lambda + l)}{v_1\varepsilon + \pi_1(\lambda + l)} \tag{4.11}$$

In graphical terms, the labour supply growth schedule is coincident with the labour effectively employed growth schedule, since the first one is completely endogenous to the second. As can be seen from Figure 4.3, there is no need for adjustment and hence the economy is always in equilibrium (natural and actual growth rates do not diverge). In this case, again, demand accommodates supply, and the growth rate of an economy is *fully-demand determined*.

Figure 4.3 – Adjustment when labour supply is completely endogenous to demand



4.3. Reconciling McCombie and Palley-Setterfield reviewed approaches

Although the results of these two adjustments are structurally different – in McCombie's approach it is always the natural growth rate that adjusts towards the BPCG rate, whilst in Palley-Setterfield approach both results are possible – the models are very similar in terms of the equations needed for their definition. The main difference is in the determination of labour supply, n, which is endogenous to McCombie (2011) and exogenous to Palley (2003) and Setterfield (2006). Equations (4.1), (4.2), (4.3b) and (3.5b) are valid in both views. Thereby, for the reconciliation, we define an equation for labour supply that encompasses both McCombie's and Palley-Setterfield's approaches.

If one assumes that income-elasticity of labour supply is linear, such as in Setterfield (2013), both approaches (Palley-Setterfield's and McCombie's) can be interpreted based on the before mentioned equation (4.12):

$$n = \gamma + \delta y \tag{4.12}$$

where $\gamma = n(0)$ and $\delta = dn(y)/dy$.

Whilst Palley (2003) and Settefield (2006) are assuming that labour supply is constant and equal to n_0, which means that $n(0) = n_0$ and $dn(y)/dy = 0$, McCombie (2011) assumes that labour supply adjusts to labour demand, and hence $\gamma = n(0) = -\lambda$ and $\delta = dn(y)/dy =$

$(1-v)$. In terms of the labour supply and employment growth diagram, the discussion is about the intercept and the slope of the labour supply curve.

Equation[14] (4.12) can replace either equation (4.4) and equation (4.9b), and a and b define whether the McCombie's approach or Palley-Setterfield approach are valid. Replacing (4.12), (4.3b) and (4.5b) in $\dot{E} = Ee$, which is given by the definition of $e = l - n$, we have that:

$$\dot{E} = E[(1 - v_0 - v_1 E)y - \lambda - \gamma - \delta y] \tag{4.13}$$

If one assumes, to simplify, that actual output growth is equal to long-term demand growth, $y = y_B$, equations (4.1) and (4.2) and (4.13) are enough to define the general model, which encompasses either McCombie's (2011) and Palley-Setterfield (Palley, 2003; Setterfield, 2006) approaches. The full representation of the model is detailed in the Appendix B, including the solutions for steady state and stability. The values of δ and γ also impacts the employment equilibrium value, which is given by:

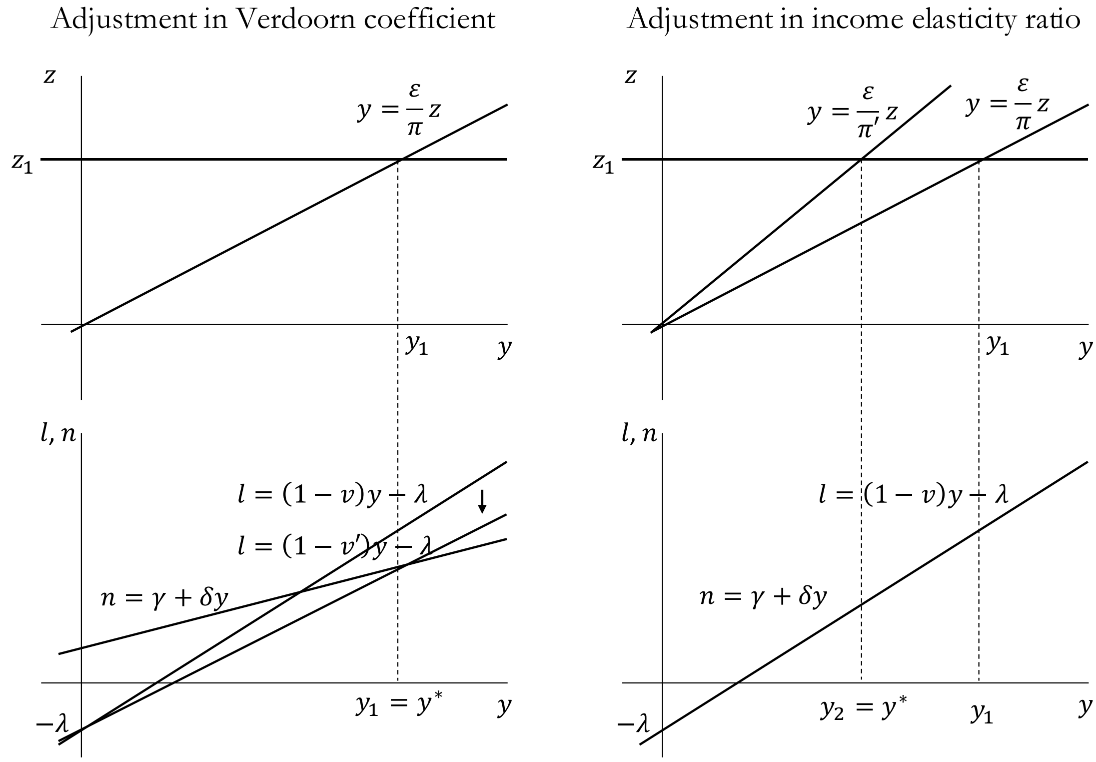
$$E^* = \frac{(1-v_0)\ \varepsilon z - \delta \varepsilon z - (\lambda+\gamma)\pi_0}{(\lambda\pi_1 + \gamma\pi_1 + v_1 \varepsilon z)} \tag{4.14}$$

Now, based on equation (4.12), we represent an intermediate case, where neither labour supply is exogenous, nor it is completely endogenous to its demand. The interesting point about this intermediate case is that there are evidences of its endogeneity (see McCombie and Thirlwall (1994) for a discussion on that), but literature is not conclusive about it being completely endogenous as argued by McCombie (2011).

Figure 4.4 presents both Setterfield (2006) and Palley (2003) adjustments in this intermediate case. In the left-hand case, where the Verdoorn coefficient is the adjustment variable, long-term growth rate is fully-demand determined. This adjustment is very similar to the one of Figure 4.2, but labour supply also increases to accommodate its demand, and hence the Verdoorn adjustment does not need to be as large as it was before.

[14] It is relevant to mention that v depends on E, but in the McCombie case, as we are aware that E does not change, then $E_0 = E^*$. With that, we are able to define the value of δ for the McCombie case as $\delta = 1 - v_0 - v_1 E_0$.

Figure 4.4 – Adjustment of labour supply and effectively employed in the Palley-Setterfield case



The main difference is in the right-hand case, where income elasticity of demand for imports is the variable of adjustment. In this case, if one assumes a demand shock (for example in z), a complex process emerges since demand adjusts via changes in elasticities ratio, and supply adjusts via movements in the labour market – and the labour supply will respond positively to the shock.

4.4. Dynamic adjustment in supply and demand in the general case

For better understanding the consequences of the dynamic adjustment for supply and demand proposed here, we present a graphical representation of each of the cases. Figures 4.5 to 4.7 present how this dynamic adjustment takes place, considering different parameter values to determine each case.

We present nine possible cases, in groups of three. In all cases, the economy is in equilibrium when world growth, z, is equal to 4%. We considering $z = 5\%$ to simulate a positive external demand shock.[15]

[15] The simulations use the following parameters for all cases: $\varepsilon = 1.5$, $\lambda = 0$, $z = 0.05$. In the first group, $\pi_0 = 1.5$, $\pi_1 = 0$, $v_0 = 0$, $v_1 = 1$; in the second group, $\pi_0 = 1$, $\pi_1 = 1$, $v_0 = 0.5$, $v_1 = 0$; in the third group, $\pi_0 = 1$, $\pi_1 = 1$, $v_0 = 0$, $v_1 = 1$. Within the groups, the following variables are different for the labour supply: in black, $\gamma = 0$, $\delta = 0.5$; in blue, $\gamma = 0.02$, $\delta = 0$; in red, $\gamma = 0.01$, $\delta = 0.25$.

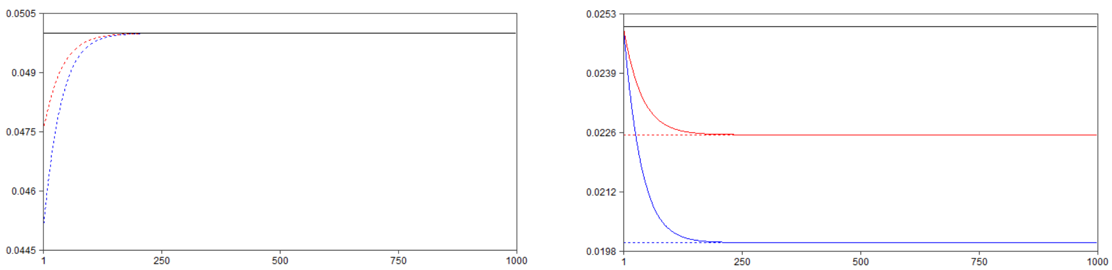
The first group, presented in Figure 4.5, considers that only the Verdoorn coefficient is endogenous to capacity utilization, as in the situation proposed by Setterfield (2006). The three cases in this group differentiate themselves for considering different labour supply schedules. The blue one considers that labour supply is exogenous, the black one considers that labour supply is completely endogenous to its demand, and the red one considers an intermediate case, where it is not exogenous but do not adjusts perfectly to accommodate its demand.

As can be seen from the left-hand side of Figure 4.5, output growth is *fully-demand determined* in all cases, as suggested before. The natural rate of growth in all cases converges to the actual growth rate, but in a different path. In McCombie's (2011) case, where labour demand accommodates labour supply, the adjustment is instantaneous. Thereby we cannot see the black dashed line (which represents the natural growth rate) as it is equal to the solid line (which represents the actual growth). However, as the labour supply became less endogenous, the time necessary for the adjustment increases.

The adjustment process can be seen in the right-hand side: in McCombie's (2011) case, represented by the black line, labour supply growth is always equal to labour effectively employed growth, and thus there the solid and the dashed lines are coincident. Conversely, if there labour supply is exogenous, a demand shock increases labour effectively employed growth, but, as Verdoorn coefficient, increases, employment growth reduces to adjust towards labour supply growth. Not surprisingly, the intermediate case provides an intermediate adjustment: the demand shock will increase labour demand and labour supply, but the effect in the first is higher than in the second. However, as time passes, since actual output growth does not change (all adjustment is in the Verdoorn coefficient), employment growth decreases and adjusts towards the new labour supply growth rate.

Figure 4.5 – Adjustment with only the Verdoorn coefficient as endogenous to capacity utilization for different labour supply schedules

Output dynamics | Labour dynamics



Dashed lines: natural rate of growth (left) and growth of labour supply (right); solid lines: actual growth rate (left) and effectively employed labour growth (right). Blue lines: exogenous labour supply; black lines: completely endogenous labour supply; red line: intermediate case.

Results become more interesting (and less predictable) when there is an adjustment in income elasticity of demand for imports, as suggested by Palley (2003). If one assumes that the Verdoorn coefficient is not endogenous, but we may face with different labour supply schedules, growth can be either *fully-supply* or *fully-demand determined*. As can be seen from Figure 4.6, if one assumes that labour supply is completely endogenous to its demand, growth is fully-demand determined, since labour supply adjusts instantaneously to it demand, and there is no change in capacity utilization.

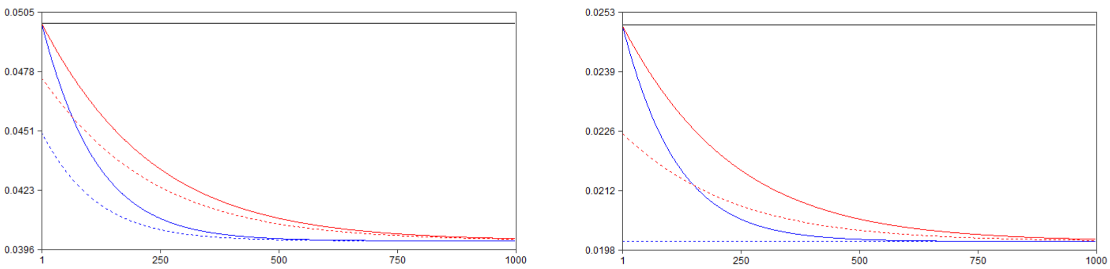
In the case of labour supply being not perfectly endogenous, growth in the long run is fully-supply determined. In the other extreme case, where it is exogenous, one could expect this result, since the labour effectively employed growth will have to adjust to labour supply growth as the only adjustment mechanism is the income elasticity, and hence the actual growth rate. Labour effectively employed adjusts towards its supply (which is given), and the economy returns to an equilibrium where the actual growth is independent of demand dynamics.

The intermediate case, however, is the most interesting and the one that can bring some new elements to the debate. The demand shock will increase both the actual and the natural rate of growth. However, the actual growth rate will be higher than the natural growth rate, once the Verdoorn coefficient is lower than one (the impact of y on y_N is lower than the unity). Labour supply growth is also lower than labour effectively employed as the adjustment is not complete. This fact causes employment rate (capacity utilization) to increase, and, consequently, the income-elasticity of demand. As a consequence, actual growth rate will decrease, reducing both labour effectively employed and labour supply growth rates. In the long run, when the new equilibrium is reached, growth rate returns to its original state (before the demand shock), which means that the economy is *fully-supply determined* even though labour supply is endogenous.

Nevertheless, we have to observe the time necessary to the adjustment. In this case, as can be seen from Figure 4.6, this adjustment can take years many time periods. Moreover, since it takes so long for the adjustment takes place; one could expect that hysteresis effects could emerge, and the supply side of the economy to be affected. One possible impact could be an increase in R&D investments and other aspects that may change the exogenous technological change, λ, or even the elasticity of labour supply to output, which means that growth can be demand determined in the long run.

Figure 4.6 – Adjustment with only import elasticity as endogenous to capacity utilization for different labour supply schedules

Output dynamics　　　　Labour dynamics



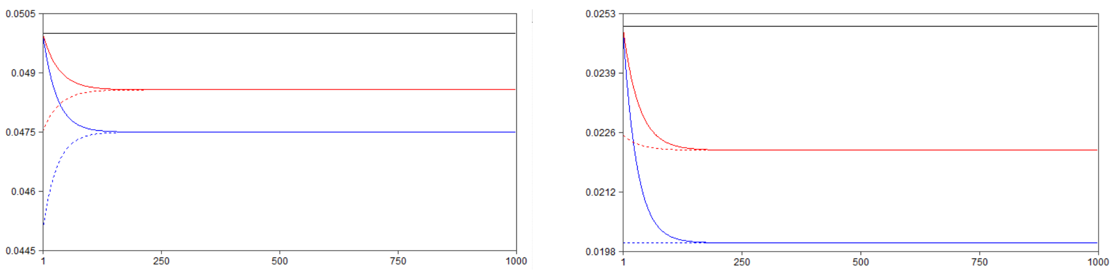
Dashed lines: natural rate of growth (left) and growth of labour supply (right); solid lines: actual growth rate (left) and effectively employed labour growth (right). Blue lines: exogenous labour supply; black lines: completely endogenous labour supply; red line: intermediate case.

Finally, the last group of cases we simulate is those in which both the Verdoorn coefficient and the income elasticity of demand for imports are endogenous to capacity utilization. Figure 4.7 presents the results for this group of cases. As can be seen in the left-hand graph, growth can be fully-demand or partially demand-partially supply determined depending on the parameters. In the extreme case, where McCombie's (2011) adjustment takes place (labour supply is completely endogenous), growth is fully demand-determined, as in all other groups of cases. Conversely, when it is completely exogenous, convergence occurs in an intermediate case, where both demand and supply forces are relevant to explain growth dynamics. In this case, labour supply growth is given, and labour demand adjusts towards it. However, during this process, employment rate (or rate of capacity utilization) increases and both the Verdoorn coefficient and the income elasticity of demand for imports also increases. Consequently, the actual and the natural growth rates will move in opposite directions. The actual growth rate, which had grown due to demand shock, will reduce, whilst the natural growth rate, which had also grown but less than the actual growth rate, will increase. In this sense, they will converge to an intermediate case.

By analysing the dynamics of labour supply, some other results emerge. As can be seen from the red lines in the right-hand side graph, the positive shock on demand will increase the labour supply, but not enough to reach the labour effectively employed. Therefore, employment rate will increase, and both the Verdoorn coefficient and the import elasticity will increase. This have negative impacts on the actual growth rate, and, consequently, labour supply will decrease. Labour effectively employed will decrease as well, since Verdoorn is increasing and demand is decreasing. However, it will decrease faster than labour supply growth rate, and thus they converge. As can be seen in the left-hand graph, actual and natural growth rates converge, but to a higher level than the case where labour supply is exogenous. From that one can conclude that growth is *partially-demand* and *partially-supply* determined. Moreover, the faster the labour supply adjusts to its demand, more growth is demand determined.

Figure 4.7 – Adjustment with both the Verdoorn and the import elasticity as endogenous to capacity utilization for different labour supply schedules

Output dynamics | Labour dynamics



Dashed lines: natural rate of growth (left) and growth of labour supply (right); solid lines: actual growth rate (left) and effectively employed labour growth (right). Blue lines: exogenous labour supply; black lines: completely endogenous labour supply; red line: intermediate case.

The value of δ, which measures the labour supply elasticity to output, is a key variable on understanding whether growth is demand or supply determined, such as presented by

McCombie (2011) and Setterfield (2013). However, only looking at this variable is not enough to understand the dynamics of supply and demand. With the aim of understanding the dynamic adjustment of actual and natural growth rates, we also need to consider the adjustment issues discussed by Palley (2003) and Setterfield (2006). If labour supply does not adjust completely to its demand, different results can be obtained for different adjustments of the Verdoorn and the income elasticity of demand for imports. These results are different not only in terms of the stable equilibrium, but also in terms of the time necessary to reach it.

These three groups of cases summarize each of the cases we present in the debate. The contribution to the literature resides in a proposal for reconciliation of the debate about the convergence between supply and demand growth rates.

5. Conclusion

In this paper we present the state of the current debate in terms of the convergence between supply and demand in Kaldorian models. We raise the literature on the different adjustment propositions between the natural rate of growth and the effective rate based in the Palley (2003) and Setterfield (2006) debate, followed by McCombie's (2011) critique. Then we follow the answer by Setterfield (2013), and his considerations on growth adjustments under capital and labour constraints. We propose a critique to his vision of capital constraints, showing that there is no need for reconciliation if firms invest to keep the rate of capital utilization unchanged. However, Setterfield's (2013) discussion on labour constraints brought some important issues to the debate, and hence there is the need for reconciliation due to the possibility of the emergence of labour constraints.

Our contribution to the debate goes in accordance with the classical argument of Cornwall (1977), summarised by McCombie and Thirlwall (1994), to whom it is central to analyse not only developing economies, but also advanced economies as "dual economies". In this sense, the growth of labour supply responds to the growth of wages and output, instead of exogenously given.

In order to reconciliate the different perspectives, we analyse the adjustment on employment through the dynamic behaviour of labour supply and effective labour. We propose an interpretation by modelling the labour market, following Setterfield (2013), offering a general model capable of summarizing the Palley-Setterfield (Palley, 2003; Setterfield, 2006) and the McCombie (2011) perspectives as limit cases of the same general model. This is an initial approach that shows that the faster labour supply adjusts to changes in economic growth (and to labour demand), the closer we leave a Palley-Setterfiled's result towards the McCombie's result. We argue that this model can reproduce intermediate results, based on the speed in which the two growth rates adjust.

From simulations we found that growth is always *fully-demand determined* either if labour supply is completely endogenous to its demand or if there is no adjustment in income elasticities of import. However, the adjustment processes are different in these cases. While in the case of completely endogenous labour supply all adjustment occur in this variable, and in the case of exogenous labour supply all adjustment is on Verdoorn coefficient, in the intermediate case both variables adjust for natural growth rates to adjust towards actual growth rate.

The main results, however, appears when income elasticity of demand for imports is endogenous. If it is the case and the Verdoorn coefficient is not sensible to capacity utilization, growth is *fully-demand determined* only if labour supply is completely endogenous to its demand. In all other cases (if, for example, it is endogenous but not completely), growth in the long run is *fully-supply determined*. This result, however, cannot be interpreted without considering the time necessary for the adjustment. The higher is the sensibility of labour supply to output, the slower is the adjustment. If one considers the parameters used in the simulation, the convergence can take a long time period (which could be decades or centuries). Thereby, one cannot ignore that supply can change substantially during the adjustment process. If, for example, higher actual growth rates increase investment in R&D, other variables can adjust, such as the exogenous technological progress (λ).

Another important result arises when both Verdoorn coefficient and import elasticities are endogenous to capacity utilization. In this case, the higher is the sensibility of labour supply to output, more the economy is demand determined. In the case of exogenous labour supply, it is *partially-supply* and *partially-demand determined*; in the case of completely endogenous labour supply growth is *fully-demand determined*. In contrast with the case above mentioned (in the last paragraph), in the intermediate case it is not the time necessary for the adjustment that changes, but the growth rate in the long run. The close we are to a complete endogenous labour supply, the more growth the in the long run is demand determined.

The baseline model we propose open the floor for different types of expansions, such as endogenizing technological progress through the variable λ; adding structural change, through changes in the income elasticity ratio and in the Verdoorn coefficient; supply shocks and demand shocks, such as a foreign crisis (reducing international demand).

Finally, this is a piece of a puzzle to be further developed in the Kaldorian literature.

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List of Variables

y	Effective growth rate	ε	Income elasticity of demand for exports
y_B	BOP Constrained Growth Rate	π	Income elasticity of demand for imports
y_N	Natural growth rate	π_0	Autonomous part of the income elasticity of demand for imports
E	Employment level	π_1	Sensitivity of the BOP constrained growth rate to the rate of capacity utilization.
e	Employment growth rate	z	Foreign GDP growth rate
N	Total labor supply	v	Kaldor-Verdoorn coefficient
n	Growth of labor supply	v_0	Autonomous part of the Kaldor-Verdoorn coefficient.
L	Total labor demand	v_1	Sensitivity of the Kaldor-Verdoorn coefficient to the rate of capacity utilization.
l	Growth of labor demand	λ	Autonomous productivity growth
δ	Labor-elasticity to output	γ	Exogenous growth of labor
a	Labor-output ratio	b	Capital-output ratio

Appendix A: The actual rate of growth

We have considered in this paper that the actual and the BPCG growth are the same. It was very useful to illustrate the problem of adjustment between the supply and demand sides. However, although the BPCG rate is an attractor to the actual growth rate, there is nothing which states that these growth rates are the same[16]. Deviations of the actual rate of growth from the equilibrium can be generated by a range of factors, such as the increase of government expenditures, the growth of propensity to consume, and many others. Some of these impacts increases the actual rate of growth together with the degree of capacity utilization, others do not have the same result. So that, we are going to analyse the dynamics of adjustment of the actual rate of growth, taking into account the BPCG rate and the natural rate of growth above considered.

First we have to evaluate again the dynamics of the natural rate of growth, given that y is not the same as y_B:

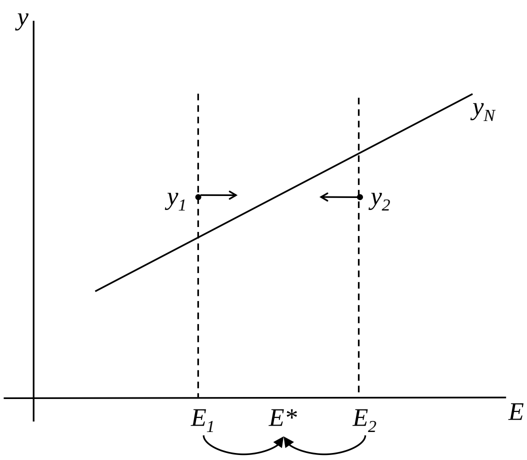
$$\frac{\partial y_N}{\partial E} = \frac{\partial y_N}{\partial \omega^K}\frac{\partial \omega^K}{\partial E} + \frac{\partial y_N}{\partial y}\frac{\partial y}{\partial E} + \frac{\partial y_N}{\partial l}\frac{\partial l}{\partial E} + \frac{\partial y_N}{\partial \lambda}\frac{\partial \lambda}{\partial E} =$$

$$= (v^K - v^C)\, y\, \frac{d\omega^K}{du} + [v^C + \omega^K(v^K - v^C)]\frac{\partial y}{\partial u} + \frac{dl}{du} + \frac{d\lambda}{du} \quad \text{(A.1)}.$$

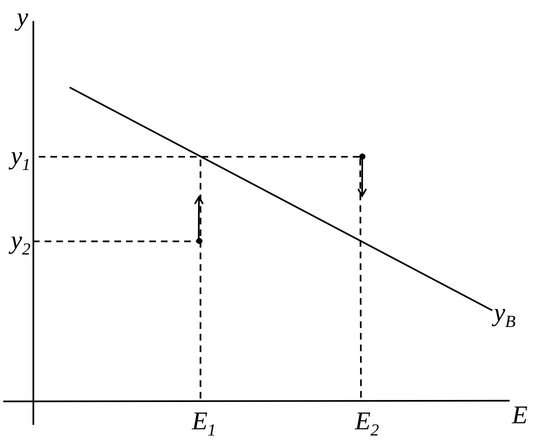
The only difference now is that $\frac{\partial y}{\partial E}$ is positive, once an increase of the degree of capacity utilization increases the actual rate of growth. Thereby, $\frac{\partial y_N}{\partial E} > 0$, which is an assumption of the paper. The BPCG rate, however, is considered to do not be affected by the actual growth rate, so we consider $\frac{\partial y_B}{\partial E} < 0$, such as before.

The adjustment of the actual rate of growth towards the natural rate of growth and the BPCG is presented by the following graphs:

Adjustment towards y_N



Adjustment towards y_B



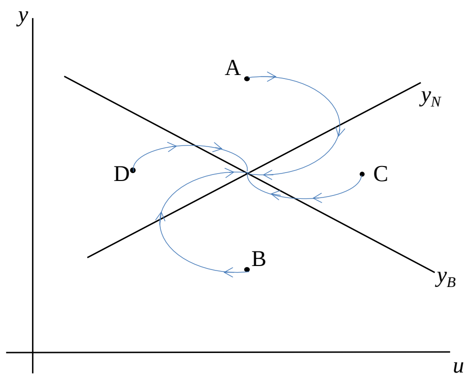
[16] Thirlwall (1979) and McCombie & Thirlwall (1994) argue that the economy tends to y_B in the long run. However, the actual rate of growth can oscillate around this rate due to a range of factors, such as changes in the terms-of-trade, inflows and outflows of capital, and many others.

The first graph shows how the actual rate of growth adjusts towards the natural rate of growth. If the economy As shown by y_1, if the actual rate of growth is higher than the natural rate of growth, the degree of capacity utilization increases $y > y_N$, $\frac{du}{u} > 0$ and thus $du > 0$. Therefore, the actual rate of growth do not change, but u moves towards u^*.

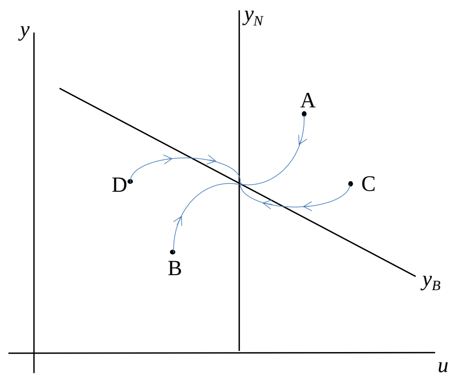
The second graph presents the adjustment of the actual rate of growth towards the BPCG rate. In this case, there is nothing *a priori* affecting the degree of capacity utilization, and thus the adjustment is only in the direction of the BPCG rate.

Taking these two dynamic processes, the process of adjustment of the actual rate of growth can be described according to the following graphs. The first considers the adjustment in the case in which the economy may be both demand and supply constrained, and the second considers the BPC case, in which the economy is only constrained by the demand side (the McCombie (2011)'s case):

Adjustment in Scenario 1



Adjustment in BPCG scenario



To understand this dynamics of adjustment let's take, for example, the circumstance which the actual rate of growth is given by the point A in the Scenario 1. Initially, as $y > y_N$, the degree of capital utilization increases, and, simultaneously, the actual rate of growth will decrease towards y_B, as $y > y_B$. However, when y finds y_N this trajectory changes: although the actual rate of growth will keep decreasing towards y_B, the degree of capacity utilization will start to decrease as the actual rate of growth tends to be lower than the natural rate of growth owing to the reduction caused by y_B.

The following table presents the logic behind these adjustment processes:

	Case 1			
	A	B	C	D
Adj. y_B	$y > y_B$, so $\dot{y} < 0$	$y < y_B$, so $\dot{y} > 0$	1) $y > y_B$, so $\dot{y} < 0$ 2) $y < y_B$, so $\dot{y} > 0$	1) $y < y_B$, so $\dot{y} > 0$ 2) $y > y_B$, so $\dot{y} < 0$
Adj. y_N	1) $y > y_N$, so $\dot{E} > 0$ 2) $y < y_N$, so $\dot{E} < 0$	1) $y < y_N$, so $\dot{E} < 0$ 2) $y > y_N$, so $\dot{E} > 0$	$y < y_N$, so $\dot{E} < 0$	$y > y_N$, so $\dot{E} > 0$

	Case BPC			
	A	B	C	D
Adj. y_B	$y > y_B$, so $\dot{y} < 0$	$y < y_B$, so $\dot{y} > 0$	1) $y > y_B$, so $\dot{y} < 0$ 2) $y < y_B$, so $\dot{y} > 0$	1) $y < y_B$, so $\dot{y} > 0$ 2) $y > y_B$, so $\dot{y} < 0$
Adj. y_N	$y < y_N$, so $\dot{u} < 0$	$y > y_N$, so $\dot{u} > 0$	$y < y_N$, so $\dot{u} < 0$	$y > y_N$, so $\dot{u} > 0$

Appendix B. Mathematical derivation of the general model

Growth Rates

The demand rate is given by the Thirlwall Law:

$$y_B = \frac{\varepsilon}{\pi_0 + \pi_1 E} z \quad \text{B.1}$$

The supply rate is given by the supply constrains ($y^N = n + q$):

$$y_N = n + \lambda + (v_0 + v_1 E)y \quad \text{B.2}$$

Labour Market

The effective labour is given by the supply condition under the effective growth rate ($l = q - y$):

$$l = -\lambda + (1 - v_0 - v_1 E)y \quad \text{B.3}$$

The labour supply is given by an exogenous parameter and sensitiveness to effective growth:

$$n = \gamma + \delta y \quad \text{B.4}$$

Adjustment

Replacing C.4 in C.2 we have that

$$y_N = \gamma + \delta y + \lambda + (v_0 + v_1 E)y \quad \text{B.5}$$

As $e = \frac{\dot{E}}{E}$ and $e = l - n$:

$$\dot{E} = E[(1 - v_0 - v_1 E)y - \lambda - \gamma - \delta y] \quad \text{B.6}$$

Defining that effective growth rate is given by demand ($\boldsymbol{y = y_B}$), then:

$$\dot{E} = E\left[(1 - v_0 - v_1 E)\frac{\varepsilon}{\pi_0 + \pi_1 E} z - \lambda - \gamma - \delta \frac{\varepsilon}{\pi_0 + \pi_1 E} z\right] \quad \text{B.7}$$

Calculating the steady state ($\dot{E} = 0$):

$$E^* = \frac{(1 - v_0)\,\varepsilon z - \delta \varepsilon z - (\lambda + \gamma)\pi_0}{(\lambda \pi_1 + \gamma \pi_1 + v_1 \varepsilon z)} \quad \text{B.8}$$

For the stability condition $\left(\frac{\partial \dot{E}}{\partial E} < 0\right)$

$$\delta < 1 - v \quad \text{B.9}$$